INTERACTIONS BETWEEN HARMONIC AND GEOMETRIC ANALYSIS

SAITAMA UNIVERSITY SATELLITE CAMPUS
TOKYO STATION COLLEGE

28 NOVEMBER – 1 DECEMBER 2016

ABSTRACTS

MINICOURSES

Ioan Bejenaru (University of California, San Diego)

Multilinear restriction theory

I will introduce the linear and multilinear restriction theory and the relation with various fields of mathematics. I will present some new results in the field with an emphasis on the role of the underlying geometry in this problem.

Anthony Carbery (University of Edinburgh)

Multilinear Kakeya-like inequalities

We survey progress over the last decade on multilinear Kakeya inequalities and some of their close cousins, focusing on obtaining sharp results. We begin by considering the Loomis–Whitney inequalities, and then some general methods for establishing multilinear inequalities. Next we will see how algebraic topological considerations come into play in the context of Kakeya-like inequalities, leading to Guth's endpoint multilinear Kakeya inequality, and, if time permits, Zhang's recent extension to k_j-planes and multilinear perturbed Brascamp–Lieb inequalities.

INVITED LECTURES

Stefan Buschenhenke (University of Birmingham)

Restriction estimates for surfaces of finite type

In the recent decades, there has been a lot of progress on restriction estimates for surfaces with non-vanishing Gaussian curvature. We present a new restriction estimate for a certain class of two-dimensional surfaces of finite type, where the curvature vanishes at some points, but they still have finite order of contact with tangent planes. We discuss some other results in this setting and present a connection to classical restriction estimates.

This is joint work with Ana Vargas and Detlef Müller.

Timothy Candy (Universität Bielefeld)

Bilinear restriction estimates and applications

The bilinear restriction problem was originally developed to make progress on the question of L^p estimates for the Fourier transform of compact hypersurfaces. Bilinear restriction estimates can also be stated in terms of products of transverse waves, and thus are closely connected to problems in dispersive differential equations. In this talk, we will present new bilinear restriction estimates which extend estimates of Bourgain, Wolff, and Tao (among others), to critical function spaces. As an application, we show how these new bilinear estimates can be used to give a small data global well-posedness and scattering result for the Dirac–Klein–Gordon system in scale invariant function spaces.

This is joint work with Sebastian Herr.

Denny Ivanal Hakim (Tokyo Metropolitan University)

Complex interpolation for the Morrey space $\mathcal{M}^p_q$: The case $0 < q \le p < \infty$

In this talk, we discuss the quasi-normed case of the complex interpolation of the Morrey space $\mathcal{M}^p_q$ where $0 < q \le p < \infty$. We show that the first complex interpolation space $[\mathcal{M}^{p_0}_{q_0}, \mathcal{M}^{p_1}_{q_1}]_\theta$ is a subset of the Morrey space $\mathcal{M}^p_q$ and it contains a proper subspace of $\mathcal{M}^p_q$. We also prove that the second complex interpolation space $[\mathcal{M}^{p_0}_{q_0}, \mathcal{M}^{p_1}_{q_1}]^\theta$ coincides with the Morrey space $\mathcal{M}^p_q$. Our results can be seen as an extension of the case $1 \le q \le p < \infty$, proved by Lemarié-Rieusset [1] and Lu et al. [2].

Joint work with Yoshihiro Sawano (Tokyo Metropolitan University).

[1] P. G. Lemarié-Rieusset, *Erratum to: Multipliers and Morrey spaces*, (2014).
[2] Y. Lu, D. Yang, and W. Yuan, *Interpolation of Morrey Spaces on Metric Measure Spaces*, Canad. Math. Bull. 57, 598–608, (2014).

Seheon Ham (Korea Institute for Advanced Study)

Averaged decay estimates for Fourier transforms of measures

For a positive Borel measure with compact support, we consider L^2 average of its Fourier transform. When the average is taken over the unit sphere, the decay estimates were studied extensively, in connection with the Falconer's distance set problem.

In this talk, we deal with average estimate over space curves with non-vanishing torsion and truncated cones. We extend the previous known results for the unit circle and the cone in three dimensions to higher dimensions. Also we discuss sharpness of the estimates and an application to the wave equation.

This talk is based on joint works with Yutae Choi (Pohang University of Science and Technology), Sanghyuk Lee, Chu-Hee Cho (Seoul National University).

Yehyun Kwon (Seoul National University)

Uniform Sobolev inequalities for second order non-elliptic differential operators

We study Sobolev-type inequality for non-elliptic differential operators of second order which is uniform in the lower order terms. We obtain a complete characterization of the Lebesgue spaces on which such estimate holds. As a consequence the result extends the class of functions in a uniqueness problem.

This is a joint work with Eunhee Jeong and Sanghyuk Lee.

Shuji Machihara (Saitama University)

Ill-posedness results for the 1d Dirac–Klein–Gordon system

We consider the Cauchy problem for the Dirac–Klein–Gordon system (DKG):

$$\text{(DKG)} \quad \begin{cases} (i\gamma_0\partial_t + \gamma_1\partial_x)\psi + m\psi = \phi\psi, \\ (\partial_t^2 - \partial_x^2 + M^2)\phi = \psi^\dagger\gamma^0\psi, \\ \psi(0,x) = \psi_0(x),\ \phi(0,x) = \phi_0(x),\ \partial_t\phi(0,x) = \phi_1(x), \end{cases}$$

where $\psi = \binom{\psi_1}{\psi_2} : \mathbb{R}^{1+1} \to \mathbb{C}^2$ and $\phi : \mathbb{R}^{1+1} \to \mathbb{R}$ are unknown functions of $(t,x) \in \mathbb{R}^{1+1}$, $\psi_0 = \binom{\psi_{0,1}}{\psi_{0,2}} : \mathbb{R} \to \mathbb{C}^2$ and $\phi_0, \phi_1 : \mathbb{R} \to \mathbb{R}$ are given functions of $x \in \mathbb{R}$, m and M are nonnegative constants, and γ_0, γ_1 are 2×2 Hermitian matrices satisfying the anticommutation relations which reads $(i\gamma_0\partial_t + \gamma_1\partial_x)^2 = (-\partial_t^2 + \partial_x^2)I_2$, where I_2 is the 2×2 identity matrix.

We discuss the time local well-posedness for DKG in the usual Sobolev spaces $(\psi, \phi, \partial_t\phi) \in H^s \times H^r \times H^{r-1}$. It is known that DKG is well-posed for $|s| \le r \le s+1$ and $(s,r) \ne (-1/2, 1/2)$. In this talk, we give the ill-posedness results for almost all other cases. We show that maps from initial data to solutions are discontinuous.

This is based on a joint work with Mamoru Okamoto from Shinshu University, Japan.

Shohei Nakamura (Tokyo Metropolitan University)

Lorentz extension including improvement of some inequalities

Christ proved the Lorentz extension of Brascamp–Lieb inequality in an appendix of Perry's paper. But the best possible of Lorentz exponent was unknown in there. In this talk, I will report the Christ's Lorentz extension is sharp. In the same context, I will give a best possible of the Lorentz improvement of Strichartz inequality for the kinetic transport equation.

This is a joint work with Professors Neal Bez, Sanghyuk Lee and Yoshihiro Sawano.

Javier Ramos (ICMAT)

The trilinear restriction estimate with sharp dependence on the transversality

We improve the Bennett–Carbery–Tao trilinear restriction estimate for subsets of the paraboloid in three dimensions, giving the sharp factor depending on the transversality.

Hiroki Saito (Kogakuin University)

Weighted maximal operators and related topics

The Hardy–Littlewood maximal operator plays an important role for the study of real analysis and harmonic analysis. Averaging is an important operation in analysis and naturally arises in many situations. In particular, the boundedness of the Kakeya maximal operator connects with many problems of analysis, for instance, Bochner–Riesz conjecture, Fourier restriction conjecture and the conjecture of geometrical dimensions of the Besicovitch set. These areas are very active, but despite much recent progress, our understanding of the problems and their relationships to each other is far from complete.

Weight theory also arise naturally in harmonic analysis. It is well known that the Muckenhoupt A_p condition characterizes the weighted L^p estimate for several important operators. The reverse Hölder class RH_1 is closely related to the Muckenhoupt class A_∞. However, it seems that further much more is known about the Muckenhoupt class A_∞ than about the reverse Hölder class RH_1 when the base family is general open sets instead of cubes in a Euclidean space. In this talk, we summarize these topics briefly and discuss recent works of several maximal operators equipped with the reverse Hölder class of weights.

This is a joint work with Professor Hitoshi Tanaka.

Hitoshi Tanaka (Tsukuba University of Technology)

An elementary proof of the endpoint estimate for the strong maximal operator

In my talk I introduce the Fefferman–Stein type inequalities for the strong maximal operator and I will present an elementary proof of the endpoint estimate for this operator. Our method used here is a covering lemma for rectangles due to Robert Fefferman and Jill Pipher.