Design and construction of the foundations for Akashi Strait Bridge

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Location of Akashi Kaikyo (Strait) Bridge
Akashi Strait Bridge
Akashi Strait Bridge
The longest suspension bridge, but the worst ground condition ever for long suspension bridges in Japan

(*about 4 million years)
Anchorages 1A and 4A

(Unit:m)
Piers 2P and 3P

(Unit: m; and the figures in the parentheses are for 3P)
Design and construction of the foundations for Akashi Strait Bridge

A geotechnical engineering case history showing the importance of
- triaxial compression tests over unconfined compression tests to estimate the strength of gravelly soil and sedimentary soft rock for limit equilibrium stability analysis; and

- the stress-strain behaviour at very small strains and its accurate measurement to estimate the displacements of the foundations.
Design and construction of the foundations for Akashi Strait Bridge

A geotechnical engineering case history showing the importance of
- triaxial compression tests over unconfined compression tests to estimate the strength of gravelly soil and sedimentary soft rock for limit equilibrium stability analysis; and

- the stress-strain behaviour at very small strains and its accurate measurement to estimate the displacements of the foundations.
Gravel (Akashi Formation)

Diameter (mm)

Percentage passing in weight

Maiko, Kobe City to Matsuho, Awaji Island

Maiko: 960
Holocene and Late Pleistocene Deposits

Sedimentary Softrock (Kobe Group)

Granite

TP (m)

0.1 1.0 50 10

0 50 100
Platform under operation for geological survey and undisturbed sampling
A sample retrieved by large triple-tube core sampling (year 1986), 30 cm in diameter and 60 cm in height, gravelly soil from Akashi Formation; and a specimen after a cyclic undrained triaxial test.
Large triaxial apparatus;
30 cm in dia. x 60 cm h
Grading curves of the samples of Akashi gravelly soil used in undrained cyclic triaxial tests

Gravelly soil (Akashi formation) used in cyclic triaxial tests (series 2)
A typical set of CU TC tests on Akashi gravelly soil (Yamagata et al., 1995)
Design shear strength under drained and undrained condition for limit equilibrium stability analysis

Undrained for seismic design
Mohr’s circles of stress at failure of isotropically consolidated samples and failure envelopes (CD TC)
Mohr’s circles of stress at failure of isotropically consolidated samples and failure envelops (CU); and definition of shear and normal stresses along the failure plane for CU TC tests ($\tau_f$ & $\sigma_m$: shear & normal stresses along the failure plane)
Stress-strain relations from CU TC tests before and after cyclic undrained loading; and a decrease ratio of undrained TC strength by cyclic undrained loading (a small decrease)
Sedimentary soft rock (Kobe Formation)
Unconfined compression test on sedimentary soft rock, reliable?

Kobe formation (Sedimentary soft sandstone), Matsuho site:
(depth: 20.6 – 20.9 m)
ρt = 2.3 g/cm³

Axial stress (kgf/cm²)

Strains in the ground
Axial strain (%)

Specimen

Axial stress
Axial displacement

E_i ≒ E_50
First stage of investigation:

- A great amount of unconfined compression tests, following the common practice at that time

→ An extremely large scatter in the unconfined compressive strength

→ Statics analysis?

Distributions of $q_u$ of rotary core tube samples of sedimentary soft rock, classified according to the soft rock type (complied in 1985; Tatsuoka & Kohata, 1995).
First stage of investigation:

- Unconfined compressive tests

→ A very large scatter due to large effects of sample disturbance (large under unconfined condition) and confining pressure

Distributions of $q_u$ of rotary core tube samples of sedimentary soft rock, classified according to the soft rock type (compiled in 1985; Tatsuoka & Kohata, 1995).
Second stage of investigation:
A typical geotechnical profile of Kobe Formation, Matsuho site, near Anchorage 4A

→ A much larger scatter in unconfined compressive strength than in CU TC strength (by a geotechnical consultant)
Unconfined compressive strength:

- A very large scatter; and
- A much lower than CU TC strength

→ Utterly unreliable
→ Switch to CU and CD TC tests to obtain design strength

Comparison of compressive strength between U and CUTC tests of sedimentary soft rock, Matsuho site, near A4 (by Yamada, S. and Tatsuoka, F.)
Typical sets of CU TC tests on Kobe sedimentary soft rock: a) sandstone; and b) mudstone (Yamagata et al., 1995)

→ A much more serious problem with unconfined compression test in the evaluation of stiffness at small strains
Design and construction of the foundations for Akashi Strait Bridge

A geotechnical engineering case history showing the importance of
- triaxial compression tests over unconfined compression tests to estimate the strength of gravelly soil and sedimentary soft rock for limit equilibrium stability analysis; and

- the stress-strain behaviour at very small strains and its accurate measurement to estimate the displacements of the foundations.
To predict ground deformations at working loads, the elastic property is important; because:
1) strains in the ground are relatively small; and
2) deformation properties at these small strains can be linked to the elastic property,

and the elastic property in the field can be obtained by field shear wave velocity measurements.
Time-history of the settlement of Pier Foundation 2P on a gravel deposit for Akashi Strait Bridge

(Tatsuoka & Kohata, 1995; Tatsuoka et al., 2001)
- Instantaneous settlement
- Residual settlement
Creep settlement
Settlement by seismic effects

Pier 2P of Akashi Strait Bridge

End of tower construction

The 1995 Hyogo-ken Nambu Earthquake

14th Oct., 1989

10 years

Applied pressure at the footing base, \( p_{ave} \) (MPa)

Settlement, \( S_i \) (mm)

Elasped time (days)
Time-history of the settlement of Pier Foundation 3P on a sedimentary soft rock for Akashi Strait Bridge

(Tatsuoka & Kohata, 1995; Tatsuoka et al., 2001)
- Elastic component is large part of the total settlement
- An increase in the tangent modulus \( \frac{d(p)_{\text{ave}}}{dS} \) with \( (p)_{\text{ave}} \), reflecting an increase in the loading rate “\( \frac{d(p)_{\text{ave}}}{dS^e} \)” with \( (p)_{\text{ave}} \).

(Tatsuoka et al., 2001)
- Lower stiffness at slower construction rates
- Noticeable creep settlement during the cease of construction

\[ S^e (\text{elastic component, based on } V_s^{\text{field}} \text{ & its pressure-dependency from laboratory stress-strain tests}) \]

\[ S^{ir} (= S^t - S^e) \]

Fitted to \[ \dot{S}^{ir} = 0.1 \text{ mm/day} \]

\[ S^t (\text{total settlement as measured}) \]

Settlement rate

\[ \dot{S}^{ir} (\text{mm/day}) \]

- 0.10
- 0.0-0.05
- 0.05-0.10
- 0.10-0.15
- 0.15-0.20
- 0.20-0.25
- > 0.25

Pier 2P of Akashi Strait Bridge
Decomposition of settlement to elastic and irreversible components, Pier 3P (Tatsuoka et al., 2001)
- Lower stiffness at slower construction rates
- Noticeable creep settlement during the cease of construction
Centreline vertical strains in the gravel and sedimentary softrock below the piers:

Strains less than about 0.5%

(Tatsuoka et al., 2001)
A series of field and laboratory tests at the site of A1
A series of field and laboratory tests at the site of A1

- E\(_0\) average of E\(_0\) (from CD TC tests (\(\sigma_c' = 0.51\) MPa = \(\sigma_v'\) (in-situ), axial strains measured with LDTs)

- \(E_{50}\) unconfined compression tests (from external axial strains)

- \(E_{\text{initial}}\) (or \(E_{50}\)) from unconfined compression tests (from external axial strains)

- \(E_{\text{PMT}}\); Pressuremeter tests (primary loading)

- \(+\) \(\nabla\) \(E_{\text{PLT}}\); tangent modulus in primary loading

- Plate loading tests

- \(E_f\) (from shear wave velocity)

- Depth (m)

- Young's modulus, E (GPa)
Unconfined compression test on sedimentary soft rock, reliable?

Kobe formation
(Sedimentary soft sandstone), Matsuho site:
(depth: 20.6 – 20.9 m)

\(\rho_t = 2.3 \text{ g/cm}^3\)

\(E_i \equiv E_{50}\)

Axial stress

Axial displacement

Specimen

Strains in the ground

Axial strain (%)
Pressure-meter tests:
most popular field loading test to evaluate the stress-strain behaviour
at small strains

\[ \Delta p: \text{pressure applied to the bore hole wall}^* \]

\[ u_0: \text{lateral displacement at the wall face}^* \]

\[ u: \text{lateral displacement in the ground} \]

If soil is linear material…. 

\[ \frac{\Delta p}{u_0/r_0} = 2G \]
Suspension method (local up-hole method)

Local:

$$V_s = \frac{L}{\Delta t}$$
An extremely large variation in the $E$ values obtained from different field and laboratory tests; Why?
Totally different so-called static and dynamic Young’s moduli
But, the $E_{\text{max}}$ values from relevant static tests are essentially the same with those from shear wave velocities (dynamic tests).
Sedimentary soft sandstone (Kobe Formation) (CD)

\[ \sigma_h' = 0.51 \text{ MPa (CD)} \]

CD TC test on an undisturbed sample consolidated to the field effective stress with local axial strain measurement
The $E_{\text{max}}$ values from relevant static tests are essentially the same with those from shear wave velocities (dynamic tests).
To predict ground deformations at working loads, the elastic property is important; because:
1) strains in the ground are relatively small; and
2) deformation properties at these small strains can be linked to the elastic property,

and the elastic property in the field can be obtained by field shear wave velocity measurements,

but, once the strain level exceeds the quasi-elastic strain limit, the stress-strain behaviour becomes highly non-linear and viscous effects becomes significant, and the entire pre-peak stress-strain-time relation cannot be obtained only from the elastic behaviour.
Relationships between back-calculated Young’s modulus and measured ground strain and Young’s modulus values for PM tests (Tatsuoka & Kohata, 1995).

- $E_f$: Young’s modulus from field $V_s$ before construction

2P for $(p)_{ave} = 0 – 0.52$ MPa
- $\Delta 1 – 5$: gravelly soil (Akashi)
- $\bigcirc 6 – 9$: sedimentary soft rock (Kobe)

3P for $(p)_{ave} = 0 – 0.53$ MPa
- $\bigcirc 1 – 6$: sedimentary soft rock (Kobe)
- $\bigcirc 7$: granite
- *) $E_f$ was estimated as $5 \times E_{PMT}$

$E_{PMT}/E_f$ (strains for $E_{PMT}$ unreported)

Measured ground vertical strain: $\varepsilon_1$ (%)
$E_f$: Young’s modulus from field $V_s$ before construction

$E_{\text{FEM}} / E_f$ (strains for $E_{\text{PMT}}$ unreported)

$E_{\text{PMT}} / E_f$ (strains for $E_{\text{PMT}}$ unreported)

range for soft rock

Gravelly soil

Young’ modulus back-calculated by FEM

(Tatsuoka & Kohata, 1995)
E_f: Young’s modulus from field V_s before construction.

2P for \( (p)_{ave} = 0 – 0.52 \) MPa
- 1 – 5: gravelly soil (Akashi)
- 6 – 9: sedimentary soft rock (Kobe)

3P for \( (p)_{ave} = 0 – 0.53 \) MPa
- 1 – 6: sedimentary soft rock (Kobe)
- 7: granite

*) \( E_f \) was estimated as 5x\( E_{PMT} \)

- Obvious non-linearity (mostly due to strain-non-linearity).

Measured ground vertical strain: \( \varepsilon_1 \) (%)
$E_f$: Young’s modulus from field $V_s$ before construction

2P for $(p)_{ave} = 0 - 0.52 \text{ MPa}$
- 1 - 5: gravelly soil (Akashi)
- 6 - 9: sedimentary soft rock (Kobe)

$E_f$: Young’s modulus from field $V_s$ before construction
- 1 - 6: sedimentary soft rock (Kobe)
- 7: granite

*) $E_f$ was estimated as $5 \times E_{PMT}$

$E_{PMT}/E_f$ (strains for $E_{PMT}$ unreported)

$E_{FEM}/E_f$, $E_{PMT}/E_f$ (strains for $E_{PMT}$ unreported)

- $E_{FEM}/E_f$ approaches 1.0 as the strain becomes very small.
2P for $(p)_{ave} = 0 - 0.52 \text{ MPa}$
- $\Delta 1 - 5$: gravelly soil (Akashi)
- $\bigcirc 6 - 9$: sedimentary soft rock (Kobe)

3P for $(p)_{ave} = 0 - 0.53 \text{ MPa}$
- $\bigcirc 1 - 6$: sedimentary soft rock (Kobe)
- $\bigcirc 7$: granite

*) $E_f$ was estimated as $5 \times E_{PMT}$

Very small values of $E_{PMT}/E_f$
Some lessons:

1) Strains operating in the ground supporting foundations allowing limited displacements; generally small, lower than about 0.5 %.

2) Non-linear behaviour even at strains less than 0.1 %.

3) The $E$ value approaches $E_0$ from $V_s$ as the strain approaches about 0.001 %, indicating the importance of elastic property for the prediction of ground deformation and structural displacements.

4) $E_{PMT}$ from conventional pressure-meter tests with linear interpretation could be too small to directly use in the prediction.
What is the ideal laboratory stress-strain test?

What is the difference between the ideal and actual laboratory stress-strain tests?

What is the consequence from errors in the laboratory stress-strain test?
Basic postulate:

If we can predict accurately the stress-strain-time behaviours of all the concerned soil elements in the ground, we can predict accurately the load-displacement-time behaviours of the ground and foundation.

However, the actual situation is different ......
Ideal laboratory stress-strain test (e.g., TC test) *
- local strain measurement
- high-quality core sample
- anisotropic re-consolidation as in the field
- sustained loading

* Very difficult to perform

Behaviour in the field (with creep deformation)

Stress path in the field soil element

Nearly elastic behaviour immediately after the start of loading
TC test (non-ideal)
- local strain measurement
- high-quality core sample
- isotropic reconsolidation
- monotonic loading to failure

Soil element

Difference between the field behaviour and the laboratory test result

Behaviour in the field (with creep deformation)

Stress

Strain
TC test (non-ideal)
- external strain measurement
- high-quality core sample
- isotropic reconsolidation
- monotonic loading to failure

Behaviour in the field
(with creep deformation)

Effects of bedding error
TC test (non-ideal)
- local strain measurement
- more-or-less disturbed sample
- isotropically reconsolidated
- monotonic loading to failure

Behaviour in the field
(with creep deformation)

Effects of sample disturbance
TC test (*most conventional*)
- external axial strain measurement
- more-or-less disturbed specimen
- isotropic reconsolidation
- monotonic loading to failure

Behaviour in the field
(with creep deformation)
Unconfined compression test
- external strain measurement
- noticeably disturbed sample

Stress Settlement

Soil element

Behaviour in the field
(with creep deformation)

Significant underestimation of the stiffness in the field due to inadequate stress path and history, effects of bedding error and sample disturbance
In design based on an allowable footing settlement

Behaviour in the field (with creep deformation)

Significant underestimation of strength and stiffness

Too conservative design
Another problem: the link among results from laboratory and field tests and field full-scale behaviour could be missed.
Pressure – settlement relations from four PLT tests using a 60 cm dia.-rigid plate at the excavation bottom of Anchorage 1A and relations predicted by linear theories and FEM analysis (Tatsuoka et al., 1999)
Full-scale behaviour of pier 3P on sedimentary soft sandstone.

- $E = 10000 \text{ kgf/cm}^2$
- $E_{\text{PMT}}\text{ (from 3P site)} = 2890 \text{ kgf/cm}^2$
- $E_{50}\text{ (from 1A site)} = 1777 \text{ kgf/cm}^2$

The graph shows the relationship between settlement and average pressure. The measured data is compared with FEM predictions.
Full-scale behaviour of pier 3P on sedimentary soft sandstone

Equivalent linear relation

- $E = 10000 \text{ kgf/cm}^2$
- $E_{\text{PMT (from 3P site)}} = 2890 \text{ kgf/cm}^2$
- $E_{50 \text{(from 1A site)}} = 1777 \text{ kgf/cm}^2$

Settlement, S (mm)

(p) ave. (kgf/cm$^2$)
Full-scale behaviour of pier 3P on sedimentary soft sandstone

Why very linear?

The effects of the following two factors are balanced;
1) An increase in the elastic Young’s modulus $E_v$ with an increase in $\sigma_v$; &
2) A decrease in $E_{sec}$ with an increase in the strain.
Full-scale behaviour of pier 3P on sedimentary soft sandstone

-Over-estimation of the instantaneous settlement when based on the conventional methods.
Full-scale behaviour of pier 3P on sedimentary soft sandstone

- Over-estimation of the instantaneous settlement when based on the conventional methods.
- Accurate simulation by FEM based on $G_f = \rho \cdot V_s^2$
Full-scale behaviour of pier 3P

- A significant over-estimation of the instantaneous settlement when based on the conventional methods.
- Accurate simulation by FEM based on $G_f = \rho \cdot V_s^2$

(Tatsuoka & Kohata, 1995)
Some conclusions - 1:

“Deformation of ground and displacements of structures at working loads constructed on and in the ground” could be reliably predicted;

1) based on field shear wave velocities;
2) while taking into account the non-linearity by pressure and strain (including viscous property) evaluated by relevant laboratory stress-strain tests that;
   a) measure accurately stresses and strains;
   b) using high-quality core samples; and
   c) simulating the field loading history.
Some questions:

1) The elastic deformation characteristics: can be obtained *only by dynamic tests*?

2) Are statically and dynamically determined elastic deformation properties *different*?

**Static tests (monotonic or cyclic);** stress-strain properties are obtained from measured stresses and strains! (or loads and displacements)

**Dynamic tests (RC tests & wave-propagation tests);** stress-strain properties are obtained from dynamic responses!

Importance of proper understanding of the relationship between static & dynamic tests in the framework of non-linear stress-strain behaviour.
Non-linear stress-strain behaviour from “quasi-elastic behaviour at very small strains” toward “shear banding at very large strains”

Quasi-Linear elastic

Elastic-weak visco-plastic

Elastic-strong visco-plastic

(Tatsuoka & Shibuya, 1991)
Laboratory and field testing methods to evaluate non-linear stress-strain behaviour from “elastic behaviour at very small strains” toward “shear banding at very large strains” → difficult by a single test method

(Tatsuoka & Shibuya, 1991)
What are different between static and dynamic tests?

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<th>Influencing factors</th>
<th>ML or CL</th>
<th>Loading rate</th>
<th>Wave length/particle size</th>
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Popular idea about the different factors between static and dynamic tests, but which factors are important??

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### A historical review of the comparison between static and dynamic tests - I

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- **ML or CL**: Monotonic Loading (ML) or Cyclic Loading (CL)
- **Loading rate**: Slow, Fast, Very fast
- **Wave length/particle size**: Nearly infinite, Small, Large
- **Number of cycles**: ¼ cycle, Small, Large
- **Strain level**: Very small, Medium, Large
Static torsional tests starting from early 70s’ at the PWRI and 80’s at the IIS

Hollow cylindrical specimen

$\tau_{vh}$ : shear stress

Toyoura sand

(Iwasaki et al., 1977; Tatsuoka et al., 1978, 1979a&b)
Pure shear

Initial isotropic stress state:
\[ \sigma'_2 = \sigma'_a = \sigma'_r = \sigma'_t; \text{ and } \delta = 45^\circ, b = 0.5 \]
Static torsional tests

Reliably measurable strain: larger than about 0.01%

(Iwasaki et al., 1977)
Shear stress, $\tau$

Dissipated energy $\Delta W$

Peak-peak secant modulus: $G_{eq}$

Stored energy: $W$

Shear strain, $\gamma$

Non-linear stress-strain curve of actual soil

Damping ratio:

$h = \frac{1}{2\pi} \frac{\Delta W}{W}$

Equivalent linearization taking into account the strain-nonlinearity of shear modulus and damping ratio
An equivalent linear system having the same \( G_{eq} \) & \( h \) as soil (i.e., a Voigt model)

Stress-strain curve of the equivalent linear system (note a different shape from the one of actual soil)

Damping ratio:

\[
h = \frac{1}{4\pi} \frac{\Delta W}{W}
\]

Single amplitude shear strain, \( \Delta \gamma \)

\( G_{eq} \) and \( h \) depends on \( D_g \)
An equivalent linear system having the same $G_{eq}$ & $h$ as soil (i.e., a Voigt model)

Note: This linear system can be equivalent only for a sinusoidal time history of input strain. If it is assumed that the stress-strain relation of soil were rate-independent (which is not true), the viscosity coefficient of the dash pot, $h=tl/(dg/dt)$, is inversely proportional to the frequency, $f$, of the sinusoidal wave of strain. In this sense, the equivalent linearization method is approximate in nature, in particular when used for an arbitrary time history of input strain or stress. A relevant non-linear three-component model is a more rational model (for details, refer to Di Benedetto & Tatsuoka., 1997).
Dynamic torsional tests: Resonant-column test

Reliably measurable strain, smaller than about 0.01 %, is possible, but a uncontrollable large number of loading cycles is applied.

Resonant-column apparatus
(Lo Presti, D., Technical University of Torino)
Resonant-column tests

Invented by the late Prof. IIDA, Kumiji in 1930s’ at the Earthquake Research Institute, the University of Tokyo, and then ignored in Japan. Found in 1960s’ in the USA and developed mainly at the Univ. of Michigan (by the late Prof. Richart), and imported to Japan in 1970s’ by Dr Iwasaki.

Homogeneous linear isotropic elastic media
(density: \( \rho \), shear modulus: \( G \))
(Cross-section: \( A \), mass \( M = A \cdot L \cdot \rho \))
Shear wave velocity:
\[ V_s = \sqrt{\frac{G}{\rho}} \]

Fig. 1
\[ u(x = 0) = a_0 \cdot \sin \omega t \]

\[ G = \rho \cdot \left( \frac{\omega_n \cdot L}{F} \right)^2 \]
where
\[ F \cdot \tan F = \frac{M}{M_A} \]

(Tatsuoka & Silver, 1980)

Fig. 2
1) Cyclic torsional shear tests ($f = 1/10 \text{ Hz at } N = 10$); and
2) Resonant-column tests (fast, $f = 100 \text{ Hz at } N \text{ larger 5,000}$)

A good agreement between the two types of tests at shear strains around 0.01%
Laboratory and field testing methods to evaluate non-linear stress-strain behaviour from “elastic behaviour at very small strains” toward “shear banding at very large strains” are difficult by a single test method.

Development in measurements of very small stress and strain in static tests for the last 30 years.

(Tatsuoka & Shibuya, 1991)
A historical review of the comparison between static and dynamic tests - II

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A new static (monotonic & cyclic loading) torsional shear apparatus developed at the IIS

(Tatsuoka et al., 1986, Tatsuoka, 1988)
(Tatsuoka et al., 1986)
A typical test result:
shear strain rate = 0.001 %/min

(Teachavorasinskun et al., 1991a&b; Tatsuoka & Shibuya, 1991)
A typical test result:
shear strain rate = 0.001 %/min

(Teachavorasinskun et al., 1991a&b; Tatsuoka & Shibuya, 1991)
A typical test result:

shear strain rate = 0.001 %/min

Stiffness at strain less than 0.001 %; $G_0 = 1,100 \text{ kgf/cm}^2$

(Teachavorasinskun et al., 1991a&b; Tatsuoka & Shibuya, 1991)
Toyoura sand  $e_{0.05} = 0.783$
Torsional simple shear test  MTS28

Shear modulus at strains less than 0.001%;
$G_0 = 1,100$ kgf/cm²
Resonant-column tests (fast and many cycles)

Cyclic (slow & a limited number of cycles, up to 10)

RC & SC (Iwasaki et al., 1977)

New static cyclic (slow & a limited number of cycles, up to 10) (Teachavorasinskun et al., 1991a&b)

Static monotonic (Teachavorasin-skun et al., 1991a&b)

Shear modulus ratio, $G/G_0$

Shear strain, $g$

$G_0 = 700 \frac{(2.17 - e)^2}{1 + e} \cdot p^{0.5}$

(p: kgf/cm²)

Toyoura sand

(0.0001 %) - (1.0 %)
Agreement of $G$ between static (slow ML & CL) and dynamic (fast CL) tests

- **Resonant-column tests** (fast and many cycles)
- **Cyclic** (slow & a limited number of cycles, up to 10)
- **RC & SC** (Iwasaki et al., 1977)

**New static cyclic** (slow & a limited number of cycles, up to 10) (Teachavorasinskun et al., 1991a&b)

**Static monotonic** (Teachavorasinskun et al., 1991a&b)

\[ G_0 = 700 \frac{(2.17 - e)^2}{1 + e} \times p^{0.5} \]

(p: kgf/cm²)

Toyoura sand

Shear modulus ratio, $G/G_0$

Shear strain, $g$

(0.0001 %) (1.0 %)
Another example showing a good agreement between the $G$ values from static (slow ML & CL) and dynamic (fast CL & RC) torsional tests.
A historical review of the comparison between static and dynamic tests - III

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<td><strong>Wave propagation</strong></td>
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Resonant-column tests: measuring the dynamic response (i.e., the resonant frequency) of a system including the whole specimen with a relatively large wave length.

Wave velocity measurements: measuring the velocity of wave that travels through part of a media with a relatively small wave length (usually detecting the first arrived body wave of the same category).
A good agreement with a fine sand: perhaps because of a large wave shear length relative to the particle size in the BE tests.

Tested at ENTPE in Lyon (Di Benedetto)
### A historical review of the comparison between static and dynamic tests - IV

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Study on **the effects of strain rate** on the stress-strain behaviour at small strains by means of the triaxial tests:

Why the triaxial test?
1) most relevant to undisturbed samples;
2) less sophisticated and so better operational than torsional shear tests

![Triaxial Test Diagram]

**Triaxial compression:** $\sigma_v > \sigma_h$

**Triaxial Extension:** $\sigma_h > \sigma_v$

But, one serious technical problem to be overcome: *bedding error*!
Proximity transducer

Bedding error

Pressure cell

Local axial strain

Local deformation transducer

Axial strain including B.E.
Triaxial testing system for small specimens locally measuring axial strains developed at the IIS, the University of Tokyo
LDT; Local deformation transducer
(Goto et al., 1991).

- Phosphor bronze strain-gaged strip
- Heart of LDT (includes electric resistance strain gages, terminals, wiring, sealant)
- Scotch tape used to fix wire on the specimen surface
- Instrument Leadwire
- Membrane Surface
- Pseudo-hinge
- LDT
- Membrane
- Teflon tube protection
- Active e.r.s.g.
- Gage leadwire
- Terminal
- Instrument leadwire
- PB strip
- Front face (tension side)
- Back face (compression side)
Loose Toyoura sand (void ratio, $e = 0.798$)

$\sigma'_h = 49 \text{kPa}$

(Tatsuoka et al., 1995)
Loose Toyoura sand (void ratio, e = 0.798)

\[ s'_h = 49 \text{ kPa} \]

Local deformation transducer

Proximeter

Axial strain including B.E.

Triaxial compression

\[ S'_v = S'_h \]
Loose Toyoura sand (void ratio, e = 0.798)

\[ s'_h = 49 \text{kPa} \]

Proximeter

Axial strain including B.E.

LDT

Local deformation transducer

E₀: Initial Young’s modulus
\[ = 127.5 \text{ MPa} \]
Elastic-weak visco-plastic

Quasi-Linear elastic

Monotonic loading for Over-consolidated soils, and cyclic presheared soils

Drained cyclic loading

Undrained cyclic loading of saturated loose sands and N.C. soft clays

Elastic-strong visco-plastic

Shear banding

$E_0$, $E_{\text{sec}}$, $E_{\text{eq}}$

Cyclic loading

Monotonic loading

$log(\varepsilon)$, $log(\varepsilon)_S$

$E_{\text{eq}}$

How is this picture distorted by BE!

(Tatsuoka & Shibuya, 1991)
Even with a fine sand, serious bedding error.

(Tatsuoka et al., 1995)
Loose Toyoura sand

\[ s'_h = 49 \text{ kPa} \]

Cyclic loading
\( (e_i = 0.819) \)

Monotonic loading
\( (e_i = 0.798) \)

- Serious bedding error in both ML and CL tests
- Nearly the same \( E_0 \) from static ML and CL tests only when the axial strains are measured locally.
Large triaxial apparatus
Institute of Industrial Science, University of Tokyo, 1986

Dr. Goto, S.; the inventor of LDT (local deformation transducer)
Specimen (30 cm-dia. & 60 cm-high)
Gravel showing a more significant bedding error than sand

(Tatsuoka et al., 1995)
Dense well-graded gravel

\[ s'_h = 785 \text{ kPa} \]

- Serious bedding error in both ML and CL tests
- Nearly the same \( E_0 \) from static ML and CL tests only when the axial strains are measured locally.
### A historical review of the comparison between static and dynamic tests - IV

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Usually:
- static: low strain rates
- dynamic: high strain rates

Then, the effects of strain rate are significant in static cyclic tests?

At the number of loading cycle equal to 10 for the same stress amplitude

(Tatsuoka & Kohata., 1995; Di Benedetto & Tatsuoka, 1997)
Negligible effects of loading frequency not only at strains $= 0.001\%$ or less but also at larger strains during cyclic loading.

(Tatsuoka & Kohata., 1995; Tatsuoka et al., 1999b)
Despite a large difference among the loading frequencies (up to 5,000 times), essentially no effects of loading frequency.

(Tatsuoka et al., 1999b)
Despite a large difference among the loading frequencies (up to 5,000 times), essentially no effects of loading frequency. 

(Tatsuoka et al., 1999b)
Very small effects of strain rate, or $f$, on the stiffness at small strains, in particular at more isotropic stress state,

(Tatsuoka et al., 1999b)
Very small effects of strain rate, or $f$, also on the strain path at very small strains

(Tatsuoka et al., 1999b)
Very small effects of strain rate, or $f$, on the Poisson’s ratio at very small strains.
Young’s modulus and Poisson’s ratio when $\delta e_v = \delta e_1$ increase with an increase in $s_v$ and $s_v/s_h$ respectively (this issue is discussed more in detail later in this lecture).
- Small but noticeable effects of strain rate, or \( f \), on the damping ratio, \( h_0 \) (as \( h_0 \) is more sensitive to inelastic behaviour than \( E \) and \( n \).
- More effects due to more inelastic behaviour at more anisotropic stress state
Summary of $E_v$ at strains of 0.001 % or less, obtained mostly by static tests (mostly cyclic triaxial tests, and partly monotonic triaxial tests):

- hard rock cores (Sato et al., 1997)
- mortar & concrete (Sato et al., 1997)
- sedimentary softrock
- gravels and sands
- clays
New data:
Cement-mixed well-graded gravel

CTX ($\sigma'_h = 20$ kPa)
Cement $= 60$ kg/m$^3$
Compacted at $w = 5\%$
Cured for 7 days

Omae et al. (2003)
$E_v = E_0$ in the vertical direction

Hard rock core

Concrete

Mortar

Ultrasonic wave

Resonant-column

**Hard rock core & mortar:**
1. A very low dependency on the strain rate
2. With homogeneous* materials, nearly the same statically and dynamically evaluated $E_v$ values

(* compared with the wave length)

(Tatsuoka et al., 1999a&b)

Axial strain rate, $d\varepsilon_v/dt$ (%/min)
Concrete

$E_v$ from body wave velocities (P waves): higher than those evaluated statically, due likely to a high heterogeneity* of the specimens (* compared with the wave length)
Effects of heterogeneity:

Body waves that propagate faster through relatively stiffer parts are first detected.

The resonant-frequency of a system consisting of the whole specimen represents the average stiffness of the whole specimen.
A very low dependency on the strain rate
A noticeable dependency on the strain rate

- Metramo silty sand (U)
- OAP clay (U)
- Sandy gravel (D)
- Wet Chiba gravel (D)
- Saturated Toyoura sand (U)
- Air-dried Hostun sand (D)
- Vallericca clay
- N.C. Kaolin (CU TC)

Graph showing the relationship between axial strain rate and modulus of elasticity.
Higher dependency on the strain rate at lower strain rates

Lower dependency on the strain rate at higher strain rates

\( E_v (= E_0 \text{ at strains less than } 0.001 \% ) \) from cyclic undrained triaxial tests on isotropically consolidated Metramo silty sand

(Santucci de Magistris et al., 1999)
$E_v$ and $h_0$ at strains less than 0.001 %, respectively, decreases and increases noticeably with a decrease in the loading rate; showing that
1) the stress-strain behaviour at strains less than 0.001 % is not perfectly elastic; and
2) the viscous property is important even at strains less than 0.001 %.

(Santucci de Magistris et al., 1999)
The stress-strain relations at strains less than 0.001 %: not perfectly linear! - more linear at higher strain rates
Highly linear & rate-independent (i.e., elastic) stress-strain behaviour appears only at strains of the order of 0.0001 % in this particular case.

![Graph showing the relationship between Secant Young's modulus and Axial strain rate, with data points for Metramo silty sand MO03UT.]

- **Highly elastic property**
- **Quasi-elastic property**

**Metramo silty sand MO03UT**
- 3rd cycle
- \( \sigma' = 392.4 \text{ kPa} \)
- Axial strain, \( 2(\Delta\varepsilon_{v})_{sa} \)
  - \( \triangle \): 1.05x10^-6
  - \( \diamond \): 2.02x10^-6
  - \( \bigcirc \): 5.00x10^-6
  - \( \square \): 1.47x10^-5

**Graph Details**
- **Secant Young's modulus, \( E_{sec} \) (MPa)**
- **Axial strain rate, \( \dot{\varepsilon}_v \) (\%/min)**
General trends of behaviour:
1. A longer elastic (linear) zone at a higher strain rate
2. The elastic zone may disappear at very low strain rates
The shear modulus at very small strain, $G_0$, of high-quality samples is nearly the same with “$G_f$ from field $V_s$”.

(Tatsuoka et al., 1999a&b)
The shear modulus at very small strain, $G_0$, of high-quality samples is nearly the same with “$G_f$ from field $V_s$”.

$G_0$ values of more disturbed samples are noticeably lower than “$G_f$ from field $V_s$”.

**Technical problems with conventional rotary core tube (RCT) sampling method for soft rocks**

(Tatsuoka et al., 1999a&b)
Some conclusions - 2:

3) Elastic deformation characteristics can be obtained by static tests measuring very small strains.

4) Statically and dynamically measured elastic deformation properties are essentially the same with fine-grained soils (Hardin (1978) & Woods (1991) have already pointed out this fact).